SECOND WORKSHOP
ON SOLITON THEORY, NONLINEAR DYNAMICS AND MACHINE LEARNING

Varna Bulgaria, August 16, 2024 – August 21, 2024

Book of Abstracts

Organized by:
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Program of the
First Workshop on Soliton Theory, Nonlinear Dynamics and Machine Learning
August 16 – 21, 2024, Varna, Bulgaria

The online participation is accessible via https://meet.google.com/nvg-quhe-bjy

August 16 – Arrival and registration

August 17 (Saturday)

	9.00 – 9.30	Opening
R. Ivanov	9.30 – 10.20	Water waves over a variable bottom and soliton propagation

Coffee break 10.20 – 10.50

D. Doneva	10.50 – 11.40	Nonlinear evolution of black holes in modified theories of gravity
S. Mishev	11.40 – 12.20	Application of equivariant transformations in neural networks to classification of galaxy images

Lunch

S. Yazadjiev	14.00 – 14.30	Solving differential equations with physics informed neural networks
R. Bogdanovski	14.30 – 15.10	Period variability of cataclysmic star AM Herculus. New photometric observations

Coffee break 15.10 – 15.30

General discussion	15.30 – 17.30	

August 18 (Sunday)

A. Stefanov	09.00 – 09.50	Polynomial Lax pairs with a Coxeter reduction
A. S. Carstea	09.50 – 10.40	Singularity analysis and bilinear approach to some Bogoyavlensky equations

Coffee break 10.40 – 11.00

A. Isar	11.00 – 11.50	Irreversibility and dynamics of quantum correlations in open quantum systems

Lunch

T. Mihaescu	14.00 – 14.30	Dynamics of strongly coupled harmonic oscillators in Gaussian noisy channels
E. M. Prodanov	14.30 – 15.10	Mathematical Analysis of the Van der Waals Equation

Coffee break 15.10 – 15.30

General discussion	15.30 – 18.00	

August 19 (Monday)

V. S. Gerdjikov	09.00 – 09.50	Riemann–Hilbert Problems and Integrable Equations
R. Constantinescu	09.50 – 10.40	Chaffee-Infante type equations with given classes of solutions

Coffee break 10.40 – 11.00

G. G. Grahovski	11.00 – 11.50	On the N-wave hierarchy with constant boundary conditions

Lunch

E.-M. Babalic	14.00 – 14.40	Contact Hamilton-Jacobi equations and consistency of strong SRRT inflation
A.-M. Pauna	14.40 – 15.20	Generalized Dodd-Bullough-Mikhailov equations with given classes of solutions
C. Curtin	15.20 – 15.50	The Lagrangian approach to modelling waves with a linear current component

Coffee break 15.50 – 16.10

General Discussion	16.10 – 18.00	

20:00 CONFERENCE DINNER

August 20 (Tuesday)

V. Gueorguiev	09.00 – 09.50	Reparametrization Invariance and Its Relation to the Dark Energy and Dark Matter Phenomena

Coffee break 09.50 – 10.10

S. Yazadjiev	10.10 – 10.40	Photon spheres
G. Gyulchev	10.40 – 11.30	Images of a thin accretion disk around Kerr Black Hole with synchronized scalar hair

Lunch

S. Ivanov	14.00 – 14.40	Beam parameters estimation with machine learning from incomplete data sets
K. Dimitrova	14.40 – 15.10	Applicability evaluation of selected xAI methods for machine learning algorithms for signal parameters extraction
L. Markov	15.10 – 16.00	An integral formula, with an application to differential equations

Coffee break 16.00 – 16.30

General Discussion	16.00 – 18.00	

August 21 (Wednesday)

M. Todorov	09:00 - 09:50	On a Method for Solving of Equations of Mathematical Physics in Multidimension
R. Marinova	09:50 - 10:40	Variational Method for Inverse Problems in Identifying Solitary-Wave Solutions

Coffee break 10.40 – 11.00

A. Ivanova	11.00 – 11.30	Application of Soliton Theory in Business Process Analysis
H. Vekov	11.30 – 12.00	Approximate solutions to ordinary differential equations using neural networks and Monte Carlo
A. Kolev	12.00 – 12.30	Application of machine learning methods to predict and analyze the condition of rolling bearings
	12.30 – 12.50	Closing

Polynomial Lax pairs with a Coxeter reduction

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We review the derivation of non-linear evolution equations (NLEE) with a Lax operator polynomial in the spectral parameter possessing a Coxeter reduction, i.e. with coefficients belonging to some Kac-Moody algebra. We briefly review the construction of the fundamental analytic solutions, which are used to define the scattering problem for the Lax operators. We also discuss the construction of the recursion operators and the Hamiltonian formulation of the corresponding NLEE. This can be viewed as continuation of our previous works [1, 2].

[1] Gerdjikov, V.S.; Stefanov, A.A., *Riemann–Hilbert Problems, Polynomial Lax Pairs, Integrable Equations and Their Soliton Solutions*, Symmetry 2023, 15, 1933. https://doi.org/10.3390/sym15101933

[2] Stefanov, A.A., *New Types of Derivative Non-linear Schrödinger Equations Related to Kac–Moody Algebra* $A_2^{(1)}$. Dynamics 2024, 4, 81-96. https://doi.org/10.3390/dynamics4010005

Generalized Dodd-Bullough-Mikhailov equations with given classes of solutions

Alina-Maria Păuna

Department of Physics, University of Craiova, Romania

The paper will present a specific approach within the auxiliary equations method for solving nonlinear differential equations. It will allow us to determine a generalized class of equations consistent with the solutions of an auxiliary equation. More precisely, we will investigate how the Dodd-Bullough-Mikhailov equation can be generalized from the perspective of the derivative-free term to retain as solutions those of the elliptic Jacobi equation.

Application of Soliton Theory in Business Process Analysis

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Soliton theory is a powerful tool for the analysis and modeling of nonlinear dynamical systems that are often encountered in business processes. The purpose of this paper is to examine the application of soliton equations in time series analysis of various business processes, focusing on customer service. By using soliton equations such as Korteweg-de Vries (KdV) and the nonlinear Schrödinger equation (NLS), anomalies can be detected and analyzed, leading to better process control and optimization.

Application of machine learning methods to predict and analyze the condition of rolling bearings

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Rising demands for industrial machinery quality and reliability lead to automating the analysis and prediction of failures, shifting from emergency repairs to planned maintenance. The article proposes methods for the analysis of possible rolling bearing failures. Freely accessible damage-type data sources adapted for autoregressive modeling were used. Classification techniques such as logistic regression, XGBoost, and artificial neural networks are compared. The analysis is performed in the time and frequency domains, and an attempt is made to apply the Cepstrum analysis in the field of mechanical vibrations. Some anomaly detection methods are applied, including ARIMA and autoencoder. The results are consolidated in a purpose-built web-based application. In this way, a comprehensive diagnostic system is proposed that uses modern machine learning techniques and methods for detecting and classifying anomalies in rolling bearing failures. The final goal of the proposed research is to create an integrated system with high speed and increased reliability for decision-making, thereby prolonging the life of mechatronic systems and improving the production indicators, which would have a strong economic effect.

Irreversibility and dynamics of quantum correlations in open quantum systems

Aurelian Isar and Tatiana Mihaescu

National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania

The Markovian time evolution of the entropy production rate as an indicator of the irreversibility, in comparison with the correlations like Rényi-2 mutual information, Rényi-2 quantum discord and entanglement is studied, in a bipartite quantum system consisting of two coupled bosonic modes embedded in a common thermal environment. The dynamics of the system is described in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, for initial two-mode squeezed thermal states, squeezed vacuum states and coherent states [1].

[1] T. Mihaescu, A. Isar, European Physical Journal Plus 139, 82 (2024)

The Lagrangian approach to modelling waves with a linear current component

Conor Curtin[1], Rossen Ivanov[1,2]

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[2]Institute for Advanced Physical Studies, 111 Tsarigradsko Chaussee,1784 Sofia, Bulgaria

The classical approach to modelling two-dimensional fluids involves the construction of a Hamiltonian, or total energy for the fluid system. The Hamiltonian for an Eulerian fluid is well defined, and widely used. We instead transform the to a Lagrangian viewpoint, which exhibits many similarities in the case of the model equations, but also some differences. In the case of a two-dimensional fluid with linear shear and constant vorticity, the separation into potential and kinetic energy is far from intuitive. Nevertheless the Lagrangian density function may be obtained, in this case from the Hamiltonian. While the long-wave regime reproduces the well known KdV equation, the short- and intermediate long wave regimes lead to highly nonlinear and non-local evolution equations.

Images of a thin accretion disk around Kerr Black Hole with synchronized scalar hair

Galin Gyulchev

Department of Theoretical Physics, Faculty of Physics, Sofia University, Sofia 1164, Bulgaria

We study possible observable images of a thin accretion disk around rotating hairy black holes with two non-trivial time-periodic scalar fields. Such black holes are a viable alternative to the Kerr black hole, having a much more complicated geodesic structure and resulting accretion disks. We investigate how the different amounts of scalar hair around the black holes, quantified by a normalized charge, alter the direct and indirect images of the disk. Our results show that for high values of this charge, close to a boson star limit, chaotic disk images are observed with multiple small, disconnected components. For moderately large amounts of scalar hair and corresponding normalized charge, although the images still exhibit chaotic behavior, a dominant central dark region component emerges. For lower values of the normalized charge, the accretion disk images qualitatively resemble those for the Kerr black hole.

On the N-wave hierarchy with constant boundary conditions

Georgi G. Grahovski

School of Mathematics, Statistics and Sciences, University of Essex, Colchester (UK)

In this talk, we will present the direct scattering transform for the N-wave resonant interaction equations with non-vanishing boundary conditions. For special choices of the boundary values $Q_{\pm}$, we outline the spectral properties of L, the direct scattering transform and construct its fundamental analytic solutions. Then, we generalise Wronskian relations for the case of constant boundary conditions.

Finally, using the Wronskian relations we derive the dispersion laws for the N-wave hierarchy and describe the NLEE related to the given Lax operator. The results are illustrated by an example of 4-wave resonant interaction system related to the algebra $sp(4,\mathbb{C})$.

Based on a joint work with Vladimir S. Gerdjikov [1, 3].

[1] V. S. Gerdjikov, G. G. Grahovski, *On the* 3*-wave equations with constant boundary conditions*, PLISKA Stud. Math. Bulgar. 21 (2012), 217–236 [E-print: arXiv.1204.5346].

[2] V. S. Gerdjikov, G. G. Grahovski, *On the N-waves hierarchy with constant boundary conditions. Spectral properties*, Int. J. of Geom. Methods in Modern Physics (2024) (to appear) [E-print: arXiv:2403.12925]

Nonlinear evolution of black holes in modified theories of gravity

Daniela Doneva[1,2], Llibert Aresté Saló[3] and Stoytcho Yazadjiev[1,4,5]

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[5]Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 8 Acad. G. Bonchev str., 1113 Sofia, Bulgaria

Quadratic theories of gravity with second-order equations of motion provide an interesting model for testing deviations from general relativity in the strong gravity regime. However, they can suffer from a loss of hyperbolicity, even for initial data that is in the weak coupling regime and free from any obvious pathology. This effect has been studied in a variety of cases including isolated black holes and binaries. We will report results about the loss of hyperbolicity of isolated scalarized Kerr black holes in a scalar-Gauss-Bonnet theory of gravity with Ricci coupling. We find that hyperbolicity is lost when the scalar field and its gradients become large, and identify the breakdown in our evolutions with the physical modes of the purely gravitational sector. We find that at the moment when hyperbolicity is lost, the system is already well within the regime where the effective field theory treatment of the theory is no longer valid.

Mathematical Analysis of the Van der Waals Equation

Emil M. Prodanov

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191 North Circular Road, Dublin D07 EWV4, Ireland

The parametric cubic van der Waals polynomial $pV^3 - (RT + bp)V^2 + aV - ab$ is analysed mathematically and some new generic features (theoretically, for any substance) are revealed: the temperature range for applicability of the van der Waals equation, $T > a/(4Rb)$, and the isolation intervals, at any given temperature between $a/(4Rb)$ and the critical temperature $8a/(27Rb)$, of the three volumes on the isobar-isotherm: $3b/2 < V_A \leq 3b$, $2b < V_B < 3b + \sqrt{5}b$, and $3b < V_C < b + RT/p$. The unstable states of the van der Waals model have also been generically localized: they lie in an interval within the isolation interval of V_B. In the case of unique intersection point of an isotherm with an isobar, the isolation interval of this unique volume is also determined. A discussion on finding the volumes $V_{A,B,C}$, on the premise of Maxwell's hypothesis, is also presented.

Applicability evaluation of selected xAI methods for machine learning algorithms for signal parameters extraction

Kalina Dimitrova, Venelin Kozhuharov, Peicho Petkov

Faculty of Physics, Sofia University, Sofia 1164, Bulgaria

Machine learning methods find growing application in the reconstruction and analysis of data in high energy physics experiments. A modified convolutional autoencoder model was employed to identify and reconstruct the pulses from scintillating crystals. The model was further investigated using four xAI methods for deeper understanding of the underlying reconstruction mechanism. The results will be presented and discussed in detail, underlining the importance of xAI for knowledge gain and further improvement of the algorithms.

An integral formula, with an application to differential equations

Lubomir Markov

Barry University, 11300 N.E. Second Avenue, Miami Schores, FL 33161, USA

We shall prove an integral formula which is believed to be new and appears to be of some interest. The formula can be applied to establish an interesting result (also believed to be new) in the theory of differential equations.

[1] G. Birkhoff and G.-C. Rota, *Ordinary Differential Equations,* John Wiley & Sons, New York, 1989

[2] P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons, New York, 1964

[3] A.G. Kartsatos, *Advanced Ordinary Differential Equations*, Mancorp Publishing, Tampa, Florida, 1993

On a Method for Solving of Equations of Mathematical Physics in Multidimension

Michail Todorov

Institute of Mathematics an Informatics, BAS, Sofia, Bulgaria

We consider linear multidimensional evolutionary equations or the linear part of nonlinear ones. The complex structure and the presence of terms with different physical sense requires the coordinate splitting to be preceded by splitting by physical factors (processes). In contrast to the coordinate splitting this kind of splitting can be exact in some nodes and in the intervals it can be controlled. The method in question is developed in 70s of 20th century by G. Marchuk and it is applied very successfully for solving of various problems in ecology, air and water pollution, diffusion, etc. In this paper we aim to demonstrate that the splitting by physical factors is applicable and can be efficient for solving of multidimensional evolutionary problems in the optics. Without loosing of generality we pay attention to an initial-value problem of (3+1)D equations of Schrodinger kind. We split the linear part of the differential operator and the initial conditions to two consequent Cauchy problems and by using a spectral analysis and techniques we proof that the original and the resulting splitting problems are equivalent in the ends of a given interval. For constant coefficients the above assertion holds in the whole interval. In opposite case for small enough step the approximation error varies in an acceptable range. The method is applied successfully for numerical solving of (3+1)d Schrodinger equation with sign-variable group velocity as well for (3+1)d amplitude equations (of envelope), when the propagation regimes of the light pulses may be ultra-short (femtosecond). The obtained results are reliable and give good predictions for the material quantities and dynamics of the light pulses.

Contact Hamilton-Jacobi equations and consistency of strong SRRT inflation

Elena Mirela Babalic, Calin Iuliu Lazaroiu, Victor Ovidiu Slupic

IFIN-HH, Bucharest-Magurele, Romania

I discuss a class of contact Hamilton-Jacobi equations arising from a dynamical consistency condition for ”rapid turn” inflation in two-field cosmological models and explain how they can be used to construct fiducial models in which this inflationary scenario can be realized. The equations are defined on a generally non-compact Riemann surface, the role of contact Hamilton-Jacobi action being played by the volume form of the Riemannian metric on the surface. The latter is viewed as a section of the positive half-line determinant bundle L_+ of the surface. The phase space is the first jet bundle of L_+, endowed with its Cartan contact structure. The equations are proper in the sense of Crandall and Lyons and hence they can be studied using the theory of viscosity solutions. This talk is based on our recent preprint https://arxiv.org/abs/2407.19912.

Period variability of cataclysmic star AM Herculus. New photometric observations.

Rumen Bogdanovski[1] and Nikola Antonov[2]

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Cataclysmic variables are close binary systems with a white dwarf primary that actively accretes material from a low-mass main-sequence secondary star in a semi-detached form. In polars, the white dwarf’s strong magnetic field enforces synchronous rotation of the two stars, channeling the accretion flow along magnetic field lines to the magnetic poles of the primary star. This process creates an accretion column, where material reaches high temperatures and emits thermal, X-ray, and cyclotron radiation. AM Herculis, the most luminous and extensively studied polar, displays optical brightness that serves as a direct indicator of the mass transfer rate from the secondary star to the white dwarf. Its orbital period of approximately 3.1 hours, has shown variations in previous studies.This information motivated us to conduct extensive photometric observations to study the period variability of the star.

Between April 27, 2023 and July 7, 2024, we carried out 39 all-night observations, averaging 5 hours each, from an amateur observatory in Meshtitsa (Pernik, Bulgaria) using a 0.25-m Newtonian telescope, Bessel photometric filters, and an ASI533MM Pro CMOS camera. The objective was to collect data sufficient to cover at least 1.5 orbital period per night. We used the V-band photometric data because of its highest SNR. We applied Lomb-Scargle periodogram analysis of the period variation, which revealed a very stable modulation with a period of 29 days, and a false alarm probability of $< 5\%$. This stability may suggest the presence of a relatively distant third body in the system, likely a brown dwarf or a second red dwarf. Here we discuss this possibility.

Approximate solutions to ordinary differential equations using neural networks and Monte Carlo

Hristo Vekov

New Bulgarian University, 21 Montevideo str. 1618 Sofia, Bulgaria

Approximate solution to weighted residual system which arises from orthogonal methods such as Galerkin using Monte Carlo. System consists of parametric definite integrals. Additionally an approximate solution to Legendre differential equation is given using Neural Networks.

Chaffee-Infante type equations with given classes of solutions

Radu Constantinescu and Corina N. Babalic

University of Craiova, Faculty of Science, Department of Physics, 13 A.I. Cuza Street, 200585 Craiova, Romania

In the first step, we will solve the classical Chaffee-Infante equation using Riccati as auxiliary equation. Then, in the second step, we will aim to generate other nonlinear differential equations retaining rather similar solutions, that is solutions expressed as a linear combination of the Riccati solutions. This is an interesting approach that could allow the splitting of the nonlinear equations in classes of equivalence following their solutions.

Water waves over a variable bottom and soliton propagation

Rossen Ivanov
Technological University Dublin, Ireland

This talk is a continuation of the talk given at the First workshop (2023) ”Hamiltonian methods in the theory of water waves” where we introduced the Hamiltonian formulation for the water wave problem over a flat bottom and the derivation of model equations, including integrable equations which allow soliton solutions.

The presence of variable bottom also allows a Hamiltonian formulation of the problem, however the Dirichlet-Neumann operator for the fluid domain depends on the bottom topography. As a result, the model equations are with depth-dependent coefficients, which has an impact on the soliton solutions.

We analyse the possibility of soliton fission and provide some numerical examples as well.

The talk is based mostly on the paper

A. Compelli, R. Ivanov and M. Todorov, *Hamiltonian models for the propagation of irrotational surface gravity waves over a variable bottom*, Phil. Trans. Roy. Soc. A. 376, Issue 2111 (2018) Article number 20170091, http://dx.doi.org/10.1098/rsta.2017.0091; arXiv:1708.06791 [physics.flu-dyn]

Variational Method for Inverse Problems in Identifying Solitary-Wave Solutions

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Inverse and ill-posed problems are among the most active areas of research in mathematics, science, and engineering. An inverse problem is characterized by knowing the outcomes or consequences but not the underlying causes. In the context of solitary wave equations, numerical algorithms face a significant challenge because the solitary wave arises from a bifurcation, with the problem always possessing a trivial solution. This trivial solution acts as a strong attractor, making it essential to take special measures to avoid it when using iterative methods. This work introduces a novel approach to circumvent bifurcation by reformulating the solitary wave problem as an inverse problem.

Singularity analysis and bilinear approach to some Bogoyavlensky equations

Adrian Stefan Carstea

Institute of Physics and Nuclear Engineering, Dept. of Theoretical Physics 077125, Magurele, Bucharest, Romania

We discuss singularity analysis and bilinear integrability of five Bogoyavlensky differential-difference equations. Blending the singularity confinement with Painlevé property reveals strictly confining and anticonfining (weakly confining) singularity pat- terns. The strictly confining patterns are extremely useful because they provide the nonlinear substitution needed for Hirota bilinear forms and moreover, as a result of high dimensionality we have a proliferation of compatible tau functions. For the new proposed equations we get also the bilinear form and multisoliton solution, being good candidates for a new differential-difference integrable systems. In addition, using bilinear formalism we recover the integrable time-discretisations. Possible extension of the so called ”express method” which gives the algebraic entropy from singularity patterns is discussed.

Application of equivariant transformations in neural networks to classification of galaxy images

Stoyan Mishev[1,2,3] and Yoana Foteva[3]

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Equivariance is a needed property in deep neural networks [1] because the feature extraction of each layer usually must be invariant to some group of transformation of the original signal. A specific group imposes restrictions on the interlayer transformations in the network with the most prominent case of convolution being a transformation which secures translational equivariance of the signal. Our report is on the application of transformations in neural networks equivariant to the Euclidean group E(2) [3] to classification of astronomical images.

[1] Taco S. Cohen and Max Welling, *Group equivariant convolutional networks.* In Proceedings of The 33rd International Conference on Machine Learning, volume 48, pp. 2990–2999, 2016.

[2] Maurice Weiler, Gabriele Cesa, *General E(2)-Equivariant Steerable CNNs*, Advances in Neural Information Processing Systems, volume 32, pp. 2019

Photon spheres

Stoytcho Yazadjiev[1,2,3]

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[2]Theoretical Astrophysics, IAAT, University of Tübingen, 72076 Tübingen, Germany
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I will discuss some of the most important astrophysical and mathematical aspects of the photon spheres. In particular I will focus on the role of the photon spheres for the formation of the shadow of the compact objects. I will also show that, under certain conditions, the presence of a photon sphere uniquely specifies the spacetime with given asymptotic charges.

Solving differential equations with physics informed neural networks

Stoytcho Yazadjiev[1,3,4] and Daniela Doneva[1,2]

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In this talk we will discuss how physics informed networks can be used to solve differential equations describing various dynamical processes in physics. As a particular example we will show how we can use physics informed neural networks to solve the equations describing the quasinormal modes of neutron stars.

Beam parameters estimation with machine learning from incomplete data sets

Svetoslav Ivanov

Faculty of Physics, Sofia University, Sofia 1164, Bulgaria

In many high-energy physics experiments a precise knowledge of particles incident on a 2D target is crucial for understanding of the morphology of the beam source. Often only partial information of the incident particles is available due to the small sensing area, failure or missing information in some parts of the detector, etc. A Machine Learning algorithm is developed to reconstruct the parameters of the beam in cases with missing information of the event distribution. The matching of the reconstructed beam parameters by applying the developed ML algorithm on partial data to the simulated beam parameters will be presented and discussed.

Dynamics of strongly coupled harmonic oscillators in Gaussian noisy channels

Tatiana Mihaescu

National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania

We investigate the Markovian evolution of Gaussian entanglement and steering in a system consisting of two strongly coupled harmonic oscillators immersed in a structured environment. Specifically, we analyze the contribution of the interaction between modes when the magnitude of the intermode coupling strength is comparable to the local frequencies of the modes, and the rotating wave approximation does not apply. Previously, the intermode strong coupling was considered in the case of a common thermal bath [1], and presently we extend this investigation to a generalized Gaussian channel, when the environment is modeled by a collection of squeezed bosonic modes. We also provide an extended comparison of the evolution of entanglement and steering in weak and strong coupling regimes [3].

[1] J. F. Sousa, C. H. S. Vieira, J. F. G. Santos, I. G. da Paz, Coherence behaviour of strongly coupled bosonic modes, Phys. Rev. A 106, 032401 (2022)

[2] T. Mihaescu, A. Isar, Dynamics of strongly coupled harmonic oscillators in Gaussian noisy channels, (to be published)

Reparametrization Invariance and Its Relation to the Dark Energy and Dark Matter Phenomena

Vesselin Gueorguiev

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The nature of Dark Energy (DE) remains an enigma, currently hypothesized to be the source of the Universe’s accelerating expansion. This work explores the non-zero Einstein Cosmological Constant (Λ_E) as a potential manifestation of DE, interpreting it through the lens of Reparametrization Invariance (RI). The scale factor $\lambda(t)$ relevant for cosmic reparametrization, its defining equations, and their relations to Λ_E are derived. Furthermore, by insisting on a reparametrization symmetry for the equation of motion, it is demonstrated how to address the missing mass problem at galactic and extragalactic scales. This approach leads to the derivation of the Modified Newtonian Dynamics (MOND) fundamental relation $g^2 = (a0 g_N)$ within the RI paradigm, where g is the gravitational acceleration, $a0$ is the MOND fundamental acceleration, and g_N is the Newtonian gravitational constant. Remarkably, the derived values for Λ_E and $a0$ are found to be consistent with their observed orders of magnitude.

Riemann–Hilbert Problems and Integrable Equations

Vladimir Gerdjikov[1,2]

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The Lecture is based on the paper [1]. The standard approach to integrable nonlinear evolution equations (NLEE) usually uses the following steps, see [2, 3, 4]:

1. Lax representation $[L,M]=0$;
2. construction of fundamental analytic solutions (FAS);
3. reducing the inverse scattering problem (ISP) to a Riemann-Hilbert problem (RHP)
$$\xi^{+}(x,t,\lambda)=\xi^{-}(x,t,\lambda)G(x,t\lambda)$$
on a contour Γ with sewing function $G(x,t,\lambda)$.
4. soliton solutions and possible applications. Step 1 involves several assumptions: the choice of the Lie algebra g underlying L, as well as its dependence on the spectral parameter, typically linear or quadratic in λ.

Our idea is to use another approach that substantially extends the classes of integrable NLEE. Its first advantage is that one can effectively use any polynomial dependence in both L and M. We use the following steps [1]:

A. Start with canonically normalized RHP with predefined contour Γ, say $\Gamma=\mathbb{R}\cup i\mathbb{R}$
$$\xi^{+}(x,t,\lambda)=\xi^{-}(x,t,\lambda)G(x,t,\lambda), \qquad \lambda\in\mathbb{R}\cup i\mathbb{R};$$

B. Specify the x and t dependence of the sewing function defined on Γ, say by:
$$i\frac{\partial G}{\partial x}-\lambda^{2}[J,G(x,t,\lambda)], \qquad i\frac{\partial G}{\partial t}-\lambda^{4}[J,G(x,t,\lambda)]; \qquad \lambda\in\mathbb{R}\cup i\mathbb{R},$$
where J is a constant diagonal matrix.

C. Introduce convenient parametrization for the solutions $\xi^{\pm}(x,t,\lambda)=\exp(\mathcal{Q}(x,t,\lambda)$, where $\mathcal{Q}(x,t,\lambda)=\sum_{s=1}^{\infty}\lambda^{-s}Q_s(x,t)$, which is compatible with the canonical normalization of RHP.

D. This RHP gives rise to a Lax pair
$$L\psi=i\frac{\partial\psi}{\partial x}-U(x,t,\lambda)\psi(x,t,\lambda)=0, \qquad M\psi=i\frac{\partial\psi}{\partial t}-V(x,t,\lambda)\psi(x,t,\lambda)=0,$$
$$U(x,t,\lambda)=\left(\lambda^{2}\xi^{\pm}J\hat{\xi}^{\pm}(x,t,\lambda)\right)_{+}, \qquad V(x,t,\lambda)=\left(\lambda^{4}\xi^{\pm}J\hat{\xi}^{\pm}(x,t,\lambda)\right)_{+},$$
where the subscript $+$ means the polynomial part in λ of the corresponding expression. One can check that $U(x,t,\lambda)$ and $V(x,t,\lambda)$ are parametrized by by Q_1, Q_2 and their x-derivatives and give rise to a system of nonlinear evolution equations (NLEE) of nonlinear Schrödinger type.

E. Use Zakharov–Shabat dressing method to derive their soliton solutions. This requires correctly taking into account the symmetries of the RHP.

G. Define the resolvent of the Lax operator and use it to analyze its spectral properties.

These results may be generalized also to Lax pairs on homogeneous spaces. This allows one to enlarge the class of Lax pairs, to which the inverse scattering method can be applied. Obviously new classes of integrable equations could be found and their soliton solutions can be evaluated. In particular one can treat more complicated contours, such as $\Gamma = \mathbb{R} \cup i\mathbb{R} \cup \mathbb{S}^1$, where $\mathbb{S}^1$ is the unit circle. Such contours are due to additional symmetries of the Lax pair involving mappings $\lambda \to \lambda^{-1}$. Examples of NLEE with such additional symmetries will be presented.

Open problems: what type of Mikhailov reductions may be imposed on $\mathscr{Q}(x,t,\lambda)$?

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