# THIRD WORKSHOP
# ON SOLITON THEORY, NONLINEAR DYNAMICS AND MACHINE LEARNING

*Tsarevo, Bulgaria, August 18, 2025 – August 23, 2025*

## Book of Abstracts

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# Program of the Third Workshop on Soliton Theory, Nonlinear Dynamics and Machine Learning August 18 – 23, 2025, Tsarevo, Bulgaria

*The online participation is accessible via*
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*Meeting ID: 857 7714 4351 Passcode: 919132*

## August 18 – Arrival and registration

## August 19 (Tuesday)

| | | |
|---|---|---|
| | 9.30 – 10.00 | Opening |
| A. Isar | 10.00 – 10.45 | Gaussian quantum entanglement in curved spacetime |

Coffee break 10.45 – 11.00

| | | |
|---|---|---|
| T. Mihaescu | 11.00 – 11.40 | NPT entanglement witnesses in Gaussian states |
| E. Cecoi | 11.40 – 12.20 | Two-quanta features in coupled double-quantum-dot-cavity systems |

## Lunch

| | | |
|---|---|---|
| V. Ceban | 13.40 – 14.25 | Phonon assisted cavity quantum electrodynamics of double-quantum-dots |
| Md. Abdus Saad | 14.25 – 15.00 | Shaping Solitons with Complex Dispersion: Geometric and Dynamical Insights From The Modified KdV-Burgers Equation toward Optical Pulse Control |
| N. Pirovski | 15.00 – 15.30 | Soliton Theory and Nonlinear Dynamics in the Human Body |

Coffee break 15.30 – 16.00

| | | |
|---|---|---|
| General discussion | 16.00 – 18.00 | |

## August 20 (Wednesday)

| | | |
|---|---|---|
| R. Ivanov | 09.30 – 10.15 | Surface water waves interacting with vortices |
| G. Grahovski | 10.15 – 11.00 | Thermodynamic properties of quantum spin systems: scaling and disorder |

Coffee break 11.00 – 11.30

| | | |
|---|---|---|
| M. Flamarion | 11.30 – 12.10 | Solitary wave dynamics within Whitham-type equations |
| A. Stefanov | 12.10 - 12.50 | Breather and Rogue Wave Solutions of a New Three-Component System |

## Lunch

| | | |
|---|---|---|
| L. Ivanova | 14.00 – 14.30 | Modelling of Intermediate Internal Ocean Waves in the Presence of Currents and Uneven Bottom Topography |
| N. Antonov | 14.30 – 15.00 | Scientific Applications of Small-Aperture Astronomical Observatories: Case Studies in Variable Star and Exoplanet Research |
| A. Mushmov | 15.00 - 15.30 | Fighting digital fraudsters in credit lending context – an alternative approach leveraging the GA4 data |

Coffee break 15.30 – 16.00

| | | |
|---|---|---|
| General discussion | 16.00 – 18.00 | |

## August 21 (Thursday)

| | | |
|---|---|---|
| V. Gueorguiev | 09.30 – 10.15 | Reparametrization Invariant Scaling Symmetry as resolution of the Li-7 problem within the BBNS |
| E.-M. Babalic | 10.15 – 11.00 | On consistency conditions for SRRT inflation in 2-field cosmological models |

Coffee break 11.00 – 11.30 (Workshop photo)

| | | |
|---|---|---|
| G. Gyulchev | 11.30 – 12.10 | Lensing and light rings of Rotating Boson Stars with synchronized scalar hair |
| K. Staykov | 12.10 – 12.50 | Neutron stars in modified theories of gravity with scalar field |

## Lunch

| | | |
|---|---|---|
| Ts. Angelov | 14.00 – 14.30 | Effects of plasma and relativistic aberrations on the shadows of traversable wormholes |
| R. Bekir | 14.30 – 15.00 | Signatures of the dark matter halo in the polarized emission around galactic black holes |

Coffee break 15.00 – 15.30

| | | |
|---|---|---|
| General Discussion | 15.30 – 18.00 | |

## 20:00 WORKSHOP DINNER
(Tsarevo, 17 Cherkovna Str.)

## August 22 (Friday)

| | | |
|---|---|---|
| V. Gerdjikov | 09.30 – 10.15 | Riemann–Hilbert Problems and Integrable Equations |
| R. Constantinescu | 10.15 – 11.00 | Mathematical theories and models for the nonlinear proliferation of gliomas |

Coffee break 11.00 – 11.20

| | | |
|---|---|---|
| V. Videv | 11.20 – 12.00 | Characterization of Petrov manifolds using Stanilov curvature operators |
| H. Vekov | 12.00 – 12.30 | Spectral Embeddings and Multi-Metric Graph Transformations for Robust Path Computation in Large-Scale Service Provider Networks |
| L. Markov | 12.30 – 13.10 | A New Generalization of the Liouville-Jacobi Identity |

Lunch

| | | |
|---|---|---|
| C. Babalic | 14.00 – 14.40 | Integrable discretization of coupled additive Bogoyavlensky and its multiple soliton solutions |
| A. Pauna | 14.40 – 15.20 | On the use of auxiliary equations for solving nonlinear ordinary differential equations |

Coffee break 15.20 – 15.40

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|---|---|---|
| General Discussion | 15.40 – 18.00 | |

## August 23 (Saturday)

| | | |
|---|---|---|
| S. Mishev | 09.30 - 10.15 | Discovering symmetries from data by learning Lie algebras |
| A. Kolev | 10.15 - 10.45 | Enhanced Fault Diagnosis of Rolling Bearings Using LSTM Autoencoders and Feature-Based XG-Boost Classification |

Coffee break 10.45 – 11.15

| | | |
|---|---|---|
| L. Temelakiev | 11.15 – 11.45 | An Open-Source Embedded AI Vision Platform for Advanced Driver Assistance Systems |
| S. Ivanov | 11.45 – 12.15 | Beam parameters estimation with machine learning from incomplete data sets |
| Y. Foteva | 12.15 – 12.45 | Application of equivariant transformations in neural networks to classification of galaxy images |
| | 12.45 – 13.00 | Closing |

Lunch

# Breather and Rogue Wave Solutions of a New Three-Component System

Aleksander Stefanov

Faculty of Mathematics and Informatics, Sofia University, Bulgaria

We derive a new exactly solvable multi-component system of non-linear evolution equations (NLEE) related to a Lax operator, that is third order in the spectral parameter. By using the dressing method, we show that it has at least two types of non-trivial solutions: breathers and rogue waves.

## On the use of auxiliary equations for solving nonlinear ordinary differential equations

Alina-Maria Păuna

Department of Physics, University of Craiova, Romania

This report emphasizes the importance of the auxiliary equation method as an important strategy in analyzing and solving nonlinear ordinary differential equations. Auxiliary equations, in particular Riccati and Jacobi auxiliary equations, play an important role in solving nonlinear ordinary differential equations. They allow the formulation of analytic solutions by reducing complex equations to simpler and well-studied forms. Our goal is not only to use this method to obtain exact solutions, but also to extend the theoretical framework by identifying classes of equations that admit the same solution forms as the associated auxiliary equations.

# Fighting Digital fraudsters in credit lending context – an alternative approach leveraging the GA4 data

Apostol Mushmov[1,2] and D. Manjunath[1]
[1]Management Financial Group, Sofia, Bulgaria
[2] Sofia University, Bulgaria

Fighting fraudsters is one of the main tasks of the Risk Departments of NBFIs. There are many approaches and methodologies in this fight, but they all depend largely on the available data sources. Google has a digital service that provides data to its users about what is happening on their own web platforms. This data is available in real time and free of charge. In the proposed material, we present our approach on how to use this data for fraud detection and creating business value for our companies.

# Enhanced Fault Diagnosis of Rolling Bearings Using LSTM Autoencoders and Feature-Based XGBoost Classification

Atanas Kolev[1], Yassen Gorbounov[1], Hao Chen[2]

[1] New Bulgarian University, Sofia, Bulgaria

[2] China University of Mining and Technology, Xuzhou, China

Accurate diagnosis of rolling bearing faults is critical for ensuring the robustness and reliability of rotating machinery. This study presents a hybrid diagnostic framework that combines an LSTM autoencoder, trained on healthy vibration signals, with an extreme gradient boosted classifier. The features are extracted from the autoencoder's compressed signal representation and are used as inputs to an XGBoost model trained on labeled data. These features reveal underlying signal patterns that traditional methods may overlook, leading to improved fault classification. The approach achieves high detection accuracy (F1-Score and Accuracy $>$ 0.98) while reducing computational overhead by omitting the decoder during inference. Comparative results show that it significantly outperforms ARIMA-based anomaly detection. These findings support leveraging deep signal encoding to improve predictive maintenance strategies in industrial environments.

## Gaussian quantum entanglement in curved spacetime

Madalin Calamanciuc and Aurelian Isar
Department of Theoretical Physics,National Institute of Physics and Nuclear Engineering,Bucharest-Magurele, Romania

The influence of Hawking radiation on quantum entanglement for bimodal Gaussian states near a Schwarzschild black hole is investigated. It is shown that for a thermal squeezed state of a bimodal bosonic system the Hawking radiation reduces and even can destroy the entanglement between the mode of a Kruskal observer Alice and the mode of Bob, who is an accelerated observer hovering outside the event horizon of black hole. By contrary, the Hawking radiation increases and even can generate quantum entanglement between Bob and anti-Bob, who is a hypothetical observer inside event horizon. We show that in both these scenarios the competition between the contrary influences produced by Hawking temperature, squeezing and field frequency, may favour the preservation of the quantum entanglement. We investigate also the influence of the thermal environment on the behaviour in time of the entanglement between the considered observers and show that the entanglement is destroyed in a finite time for both considered bipartite scenarios of observers Alice and Bob, and respectively Bob and anti-Bob, for non-zero values of the temperature of the thermal environment, i.e. the phenomenon of entanglement sudden death takes place [1].

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1. Madalin Calamanciuc and Aurelian Isar, Results in Physics 55, 107167 (2023).

## Integrable discretization of coupled additive Bogoyavlensky and its multiple soliton solutions

Corina N. Babalic

University of Craiova, Romania

Starting from the differential-difference coupled additive Bogoyavlensky system (coupled aB) and using the Hirota bilinear formalism, we construct one completely integrable discretization. For this purpose, we consider the Hirota bilinear form of semidiscrete coupled aB, which is known to be completely integrable, and we apply the Hirota discretization technique. After checking the guage invariance for the discrete bilinear form, we build the multisoliton solutions and, as a final step, we recover the nonlinear form of discrete coupled aB, proving its complete integrability.

## Two-quanta features in coupled double-quantum-dot-cavity systems

E. Cecoi[1], T. Mihaescu[2], A. Isar[2], M. A. Macovei[1]
[1] Institute of Applied Physics, Moldova State University, Chisinau, Moldova
[2] Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest - Magurele, Romania

We shall report our recent results obtained in a complex sample consisting of a semiconductor double quantum dot (DQD), coupled in two-photon resonance to a leaking microwave resonator, whereas the inter-dot Coulomb interaction is considered strong. As a consequence, the steady-state quantum dynamics of this compound system exhibits sudden changes of its features which occur at a critical DQD-cavity coupling strength, respectively. In particular, we have shown that the electrical current through the double quantum dot follows the mean photon number into the microwave mode inside the resonator, or vice versa. Furthermore, the photon quantum correlations vary from super-Poissonian to Poissonian photon statistics, i.e. towards single-qubit lasing phenomena at microwave frequencies.

## Lensing and light rings of Rotating Boson Stars with synchronized scalar hair

Galin Gyulchev

Department of Theoretical Physics, Sofia University, Bulgaria

In recent years, there has been a growing interest in bosonic stars, both within the domain of fundamental physics and in the context of gravitational theory. These exotic compact objects can span a wide range of scales — from enormous, even surpassing galactic dimensions, to configurations comparable in size and compactness to massive black holes, rendering them potential sources of gravitational radiation. In the present work, we explore the shadows cast by rotating bosonic stars minimally coupled to a time-periodic scalar field, where the target-space geometry exhibits vanishing Gaussian curvature. Our analysis reveals distinct and clearly identifiable deformations in the surrounding spacetime, whose shape and extent vary significantly depending on the normalized charge of the underlying gravitational configurations. Moreover, we investigate the onset of chaotic features in the shadow images of extremely compact bosonic stars — a phenomenon that may bear profound observational consequences and offer new insights into the astrophysical signatures of such objects.

# Thermodynamic properties of quantum spin systems: scaling and disorder

Georgi G. Grahovski

School of Mathematics, Statistics and Sciences, University of Essex, Colchester (UK)

In this talk, we will discuss thermodynamic properties of spin-$1/2$ $XY$ antiferromagnetic Heisenberg ladders by means of the stochastic series expansion quantum Monte Carlo technique. This includes the thermal properties of the specific heat, uniform and staggered susceptibilities, spin gap, and structure factor.

We will present numerical simulations, probed over a large ensemble of random realizations in a wide range of disorder strengths $r$: from the clean ($r = 0$) case, up to the diluted ($r \to 1$) limit, and for selected choices of number of legs $L_y$ per site.

Our results show some interesting phenomena, like the presence of crossing points in the temperature plane for both the specific heat and uniform susceptibility curves which appear to be universal in $r$.

Based on a joint work with Erol Vatansever and Nikolaos Fytas [1].

*REFERENCES*

1. E. Vatansever, G. G. Grahovski and N. G. Fytas, *Thermodynamic properties of disordered quantum spin ladders*, Eur. Phys. J. B **97**(2024) 34, [E-print: arXiv.2312.12834].

## Neutron stars in modified theories of gravity with scalar field

Kalin V. Staykov[1], Peter Y. Yordanov[1], Daniela D. Doneva[2,3] and Stoytcho S. Yazadjiev[1,4]

[1] Department of Theoretical Physics, Sofia University, Bulgaria
[2] Theoretical Astrophysics, Eberhard Karls University of Tübingen, Germany
[3] INRNE - Bulgarian Academy of Sciences, Sofia, Bulgaria
[4] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria

Neutron stars offer the opportunity to probe both the properties of matter and the nature of gravity in the strong-field regime. However, uncertainties in the equation of state and in possible modifications to general relativity often lead to qualitatively and quantitatively similar observational signatures, making them difficult to distinguish. In this talk, we will focus on scalar-tensor extensions of gravity and highlight some effects that cannot be mimicked by uncertainties in the equation of state.

# A New Generalization of the Liouville - Jacobi Identity

Lubomir Markov
Barry University, USA

A generalized Liouville−Jacobi Identity is proved for the determinant $\det X(t)$ of a solution $X(t)$ to the linear nonhomogeneous first-order matrix differential equation with left- and right-coefficient matrices $\frac{d}{dt}X(t) + A(t)X(t) + X(t)B(t) = F(t), \;\; X(t_0) = X_0$.

# An Open-Source Embedded AI Vision Platform for Advanced Driver Assistance Systems

Lyuben Temelakiev and Yassen Gorbounov

New Bulgarian University, Bulgaria

Advanced Driver Assistance Systems (ADAS) such as Forward Collision Warning (FCW) and Lane Departure Warning (LDW) are widely available in commercial vehicles, yet their underlying algorithms and implementations are rarely open or accessible for academic or hobbyist use. This work presents the design and implementation of an open-source ADAS prototype running on a constrained embedded platform — the Raspberry Pi 5 equipped with a Hailo AI accelerator and standard Raspberry Pi camera module. The system integrates object detection and lane segmentation networks running in parallel on the Hailo AI processor, combined with custom real-time warning logic implemented in Python. The pipeline achieves simultaneous FCW and LDW inference at real-time frame rates without reliance on proprietary automotive hardware or software, demonstrating the feasibility of low-cost, open, and reproducible ADAS research platforms.

# Modelling of Intermediate Internal Ocean Waves in the Presence of Currents and Uneven Bottom Topography

Lyudmila Ivanova

Technological University Dublin, Ireland

A two-dimensional stratified fluid system with two incompressible and inviscid layers separated by a free interface is analyzed. The system consists of a lower medium, which includes the effects of a slowly varying bottom topography, and an upper medium with a free surface, where wind-generated surface waves occur. However, the flat surface approximation is employed based on the assumption that surface waves have negligible amplitude. In a geophysical context, this represents a model of an internal wave formed within a pycnocline or thermocline in the ocean.

Additionally, a current profile with depth-dependent currents in each domain is considered. An example of the physical situation described above is clearly illustrated by the equatorial internal waves in the presence of the Equatorial Undercurrent (EUC). Wave propagation is studied in the so-called intermediate long-wave approximation, where the wavelength is comparable to the depth of the upper layer, and the upper layer is much deeper than the lower layer.

The study is conducted using a Hamiltonian approach. The equations of motion are formulated as a Hamiltonian system, and the Hamiltonian is determined and expressed in terms of canonical wave-related variables. Applying a specific scaling leads to the integrable Intermediate Long Wave Equation (ILWE) for the flat-bottom case. For non-uniform bottom topography, the equation is generalized to include spatially varying coefficients reflecting the bottom profile. The limiting behavior of the resulting equation is investigated, and connections with other known models are established. To further enhance the completeness of the analysis, higher-order coefficients of the ILWE are also incorporated.

## Solitary wave dynamics within Whitham-type equations

Marcelo V. Flamarion[1] and Efim Pelinovsky[2]
[1]Pontifical Catholic University of Peru
[2]HSE University

Solitons are high-amplitude wave pulses that retain their shape and velocity as they travel. This unique, particle-like behavior emerges from a delicate balance between dispersion, which tends to spread the wave, and nonlinearity, which encourages steepening. By the early 20th century, the dynamics of soliton interactions in the well-known Korteweg-de Vries (KdV) equation had been extensively studied. These interactions can be described in two ways: algebraically, in terms of the ratio of initial soliton amplitudes, and geometrically, based on the number of local maxima that appear during the interaction. This classification approach was first introduced by Peter Lax. In this presentation, we explore solitary wave interactions with the Whitham equation, and its extensions. Our results indicate that, while these equations align with the geometric classification proposed by Lax, they do not permit an algebraic classification.

# On consistency conditions for SRRT inflation in 2-field cosmological models

E.M. Babalic, C. I. Lazaroiu

Department of Theoretical Physics, IFIN-HH, Magurele, Ilfov, Romania

We discuss consistency conditions for SRRT inflation for two-field cosmological models and will analyse the strong consistency condition, which can be seen as a geometric PDE which constrains the scalar field metric and potential of such models. When supplemented by appropriate boundary conditions, the equation determines the metric or the potential, one in terms of the other, selecting "fiducial" models for strong SRRT inflation. When the scalar potential is given, the equation can be simplified by fixing the conformal class of the scalar field metric. We analyze this equation with standard methods of PDE theory, discuss its quasilinearization near a non-degenerate critical point of the scalar potential and extract natural asymptotic conditions for its solutions at such points.

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[1]. E.M. Babalic, C.I. Lazaroiu, V.O. Slupic, Strong rapid turn inflation and contact Hamilton-Jacobi equations, arXiv:2407.19912v2 [hep-th].

[2]. L. Anguelova, C. I. Lazaroiu, Dynamical consistency conditions for rapid turn inflation, JCAP 05 (2023) 020, arXiv:2210.00031 [hep-th].

[3]. L. Anguelova, C. I. Lazaroiu, Consistency Condition for Slow-roll and Rapid-turn Inflation, arXiv:2311.18683 [hep-th].

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[6]. C. I. Lazaroiu, Dynamical renormalization and universality in classical multifield cosmological models, Nucl. Phys. B 983 (2022) 115940, arXiv:2202.13466 [hep-th].

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[8]. C. I. Lazaroiu, Natural observables and dynamical approximations in multifield cosmological models, Int. J. Modern Physics A. 38 (2023) 32, 2343007, arXiv:2305.04367 [gr-qc].

# Shaping Solitons with Complex Dispersion: Geometric and Dynamical Insights From The Modified KdV-Burgers Equation toward Optical Pulse Control

Md. Abdus Saad

Hajee Mohammad Danesh Science and Technology University, Dinajpur

By assembling the features of two significant nonlinear evolution equations—namely the modified Korteweg-de Vries (mKdV) and Burgers equation—the modified Korteweg-de Vries-Burgers (mKdVB) equation is derived. This nonlinear PDE plays a vital role in describing nonlinear wave propagation across diverse mathematical physics domains including optical fiber communication. In this study, we apply the improved auxiliary equation method and the improved tanh technique to obtain explicit analytical solutions of the (2+1)-dimensional mKdVB equation. The solutions exhibit kink, anti-compacton, M-shaped, and periodic structures. These patterns are graphically represented in 3D, 2D, and contour plots to reveal the effect of dissipation and dispersion. In addition to analytical solutions, a dynamical system framework is constructed to explore the evolution of wave energy with varying diffusion parameters. Through phase portraits, Hamiltonian energy functions, and time series analysis, it is shown that negative diffusion leads to energy spiraling and decay, zero diffusion results in orbital stability, and positive diffusion injects energy into the system. We also investigate perturbed diffusion coefficients (time-dependent) to simulate more realistic physical scenarios. A novel study of M-soliton interaction is included where we show how dissipation alters soliton shape, phase shifts, and trajectory. Real and imaginary diffusion coefficients are considered, corresponding to dispersive spreading and gain/loss effects in nonlinear optical fibers. This aligns with real-world applications in ultrafast optics and waveguide signal processing. The findings illustrate complex energy exchange, soliton interaction, and dissipation effects, contributing toward improved understanding of nonlinear energy transport in advanced optical systems.

## Scientific Applications of Small-Aperture Astronomical Observatories: Case Studies in Variable Star and Exoplanet Research

Nikola Antonov

Institute for Advanced Physical Studies, Bulgaria
Sofia University, Bulgaria

Over the past decades, small-aperture observatories have established themselves as capable contributors to professional time-domain astronomy. When operated remotely with modern automation, calibrated photometric filters, and high-performance detectors, they are increasingly capable of delivering results aligned with professional research standards. In this presentation, we highlight several representative results: the early detection of superhumps in the newly discovered cataclysmic variable GOTO065054+593624 during its 2024 outburst; a precisely timed transit of TOI-2109b exoplanet; and a statistical analysis of 16 B-band flares from the active M-dwarf AD Leo, including energies up to $10^{33}$ erg. These findings affirm the scientific viability of compact, autonomous observatories for capturing high-cadence data and underscore their critical role in supporting international observational networks. These case studies exemplify how compact observatories can contribute meaningfully to time-domain astrophysics, especially in monitoring dynamic phenomena often missed by large-scale surveys.

# Soliton Theory and Nonlinear Dynamics in the Human Body

Nikola Pirovski
Trakia University, Stara Zagora, Bulgaria and
Institute of Philosophy and Sociology, Bulgarian Academy of Sciences

The human body represents a dynamic system that can serve as a primary source for developing new theoretical models in biology, physics, and mostly for interdisciplinary sciences. This review integrates soliton theory and nonlinear dynamics within the Spiral Theory of the Human Body (STHB) to examine biological processes through the lens of chaotic systems. The focus is placed on morphological and functional aspects, where solitons – self-reinforcing waves that maintain their shape – explain the efficiency of neural transmission and cellular movement. Nonlinear dynamics reveals chaotic and fractal patterns in brain activity and heart rate variability, highlighting the adaptability and clinical significance of these phenomena. STHB, developed upon concepts of spiral morphology and attractors, considers the body as a system organized around a central attractor with subscales and describes the body as an autowave system. This idea is complemented by analogies with Traditional Chinese Medicine (TCM), which models the body as a network of channels and acupuncture points, similar to Ayurveda chakras, reflecting fractal and nonlinear properties. The body could be analyzed as a tensor system, emphasizing the anisotropic properties of physiological processes. The integration of the I Ching as an ancient example of fractality and synergetic principles reveals that contemporary theories are rediscovering ancient knowledge whose explanation had been partially forgotten. The review suggests that the human body is not merely an object of study, but a source of new scientific insights that connect contemporary models with philosophical heritage, guiding future research in diagnostics and therapy.

# Signatures of the dark matter halo in the polarized emission around galactic black holes

Rasim Bekir[1], Tsanimir Angelov[1], Galin Gyulchev[1], Petya Nedkova[1], Stoytcho Yazadjiev[2]

[1] Sofia University “St. Kliment Ohridski”, Bulgaria

[2] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

We study the polarization of the accretion disk around black holes within a dark matter halo. We simulate the observable polarization of a magnetized fluid ring and assess how a dark matter halo affects its characteristics, using parameters consistent with dark matter distributions observed in galaxies. For the selected sample of magnetic field configurations, the polarization intensity and direction of the direct images deviate less than 1% from the case of the Schwarzschild black hole. The deviation increases with an order of magnitude in the case of indirect images, but remains below 10% for small inclination angles, which correspond to the galactic targets Sgr A$^*$ and M87$^*$. Detection of the influence of dark matter on the polarized emission from the accretion disk remains a challenge in the near future.

## Mathematical theories and models for the nonlinear proliferation of gliomas

Radu Constantinescu
University of Craiova, Romania

The presentation will review the mathematical models proposed for describing the proliferation of gliomas, the most common brain tumors, with strong dynamic invasiveness and proliferative growth. Due to the diffuse spreading through the brain and to the heterogeneity of the tissue, the growth of bulk mass of the tumor can be mathematical described by a reaction-diffusion equation, with the unknown quantity representing the concentration of the tumor cells. The long term expansion of the tumor can be simulated as a traveling wave, solution of the considered reaction-diffusion equation. There is an interesting connection between these waves and the bifurcation theory.

# Surface water waves interacting with vortices

Rossen Ivanov[1], Delia Ionescu-Kruse[2], Michail Todorov[3]
[1] Technological University Dublin, Ireland
[2] Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania
[3] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

The study of rotational flows in inviscid, incompressible fluids with free boundaries is of significant theoretical and practical importance. This includes phenomena such as the time-evolution of surface waves in the ocean, their approximation using model equations, and the dynamics of wave interactions. Vorticity plays a crucial role in the modeling of the fluid motion.

In this relation, we examine a two-dimensional water-wave problem that includes a general non-zero vorticity field within a fluid volume that has a flat bed and a free surface. The nonlinear equations of motion in terms of the specified surface and volume variables are presented in a closed form. These equations illustrate the complex interaction between the fluid surface and the volume, indicating that a simple reduction of the model to only surface variables is not possible.

As a specific example, we analyze a point vortex and its interaction with the fluid's free surface. In the small-amplitude, long-wave Boussinesq and Korteweg-de Vries (KdV) regimes, we derive a simplified system of coupled equations that describes the motion of both the vortex and the free surface.

*REFERENCES*

D. Ionescu-Kruse, R. Ivanov, Nonlinear two-dimensional water waves with arbitrary vorticity, Journal of Differential Equations 368 (2023), Pages 317–349,
https://doi.org/10.1016/j.jde.2023.05.047

## Discovering symmetries from data by learning Lie algebras

Stoyan Mishev[1,2,3], Yoana Foteva[1]

[1] New Bulgarian University, Sofia, Bulgaria

[2] Institute for Advanced Physical Studies, Sofia, Bulgaria

[3] Varna Free University, Varna, Bulgaria

Incorporating group equivariant layers in deep neural networks is a big progress in building more explainable, simpler ML models with conrolled generalization capabilities. An obvious weak feature of this approach is that the dataset should possess some of the symmetries contained in the group. One way to build equivariant layers without prior knowledge of the symmetries in data is to learn Lie algebras. In this talk we outline how to find the Lie algebra characterizing data by training a neural network and we also discuss the link between equivariance and conservation laws.

# Beam parameters estimation with machine learning from incomplete data sets

Svetoslav Ivanov

Sofia University, Bulgaria

In many high physics experiments a precise knowledge of particles incident on a 2D target is crucial for understanding of the morphology of the beam source. Often only partial information of the incident particles is available due to the small sensing area, failure or missing information in some parts of the detector, etc. A Machine Learning algorithm is developed to reconstruct the parameters of the beam in cases with missing information of the event distribution. The matching of the reconstructed beam parameters by applying the developed ML algorithm on partial data to the simulated beam parameters will be presented and discussed.

## NPT entanglement witnesses in Gaussian states

Tatiana Mihaescu
National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania

We fully characterize entanglement witnesses based on second order statistical moments which detect negative partial transpose (NPT) entanglement in Gaussian quantum states. The NPT criterion is necessary and sufficient in 1x1 and 1xN partition of bosonic modes. We propose a semidefinite optimization algorithm that constructs an optimal NPT entanglement witness from random heterodyne measurements, and tests its efficiency in terms of number of measurements required for the certification of entanglement.

## Effects of plasma and relativistic aberrations on the shadows of traversable wormholes

Tsanimir Angelov,[1], Rasim Bekir[1], Galin Gyulchev[1], Petya Nedkova[1], Stoytcho S. Yazadjiev[1,2]

[1] Sofia University "St. Kliment Ohridski", Bulgaria
[2] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

We study the shadows of spacetime wormholes and the effects of the presence of plasma and relativistic aberrations on them as seen by a distant observer. Using the Hamiltonian approach, we determine the photon regions in which light can orbit the object and the forbidden zones through which the photons cannot travel. This shows the important effect of the density of the plasma that may surround the wormholes. Our focus is analyzing the observables, which are the shadow's shape, size, and the direction in which it can be observed. Due to the dependence of the shadow's size on the plasma density, the critical value above which it is no longer observable is explored. The analysis is based on a comparison between wormholes and Kerr black holes, as the former might be mimickers for the latter.

# Characterization of Petrov manifolds using Stanilov curvature operators

Veselin Videv
Trakia Univesity, Stara Zagora, Bulgaria

Let $(M,g)$ be a Riemannian manifold with metric tensor $g$ and curvature tensor $R$. G. Stanilov define the following curvature operators:

I. Symmetric curvature operator:

$$R_{X,Y}(u) = \frac{1}{2}[R(u,X,Y) + R(u,Y,X)]$$

, defined for any orthonormal pair $X,Y \in M_p, p \in M$.

II. Skew-symmetric curvature operator:

$$\kappa(X,Y)(u) = R(X,Y,u)$$

, defined for any orthonormal pair $X,Y \in M_p, p \in M$.

III. Generalized Jacobi operator

$$R(E^m)(u) = \sum_{i=1,m} R(u,X_i,X_i)$$

, defined for an arbitrary m-dimensional tangent subspace $E^m \subset M_p$, at a point $p \in M$, where $\{X_i\}_{i=1,m}(m < n)$ is an orthonormal basis in $E^m$;

IV. Generalized skew symmetric-curvature operator

$$\mathscr{R}(E^m)(u) = \sum_{i<j=1,m} \kappa^2(X_i,X_j)(u)$$

, defined for any $m$-dimensional tangent subspace $E^m \subset M_p$, at a point $p \in M$, where $\{X_i\}_{i=1,m}(m < n)$ is an orthonormal basis in $E^m$.

Using these curvature operators we characterize Petrov manifolds (M,g), which are decision of the Einstein equation of the Relativity:

$$\rho - \frac{1}{2}\tau g = -\lambda T,$$

, where $\rho$ is the Ricci tensor, $\tau$ is the scalar curvature, $T$ is the stress-energy tensor, $\lambda$ is a constant and $(M,g)$ is a Lorentzian manifold with metric $g$ of signature (+,+,+,-). The Petrov manifolds are 3 types depend on the curvature tensor $R$ has 1,2, or 3 eigenvalues.

## Phonon assisted cavity quantum electrodynamics of double-quantum-dots

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The phonon influence on the cavity quantum electrodynamics of a double-quantum-dot placed in an optical cavity had been investigated. We apply the reservoir theory in order to describe the interaction of a double-quantum-dot with environmental phonons. Within the framework of open quantum systems, a corresponding set of damping terms had been deduced and introduced into the master equation of the density matrix operator describing the system dynamics. These damping terms are dependent on the quanta distribution of the considered phonon environment. Here, we present and discuss the changes introduced by the phonons in the quantum dynamics of the considered model.

# Reparametrization Invariant Scaling Symmetry as resolution of the Li-7 problem within the BBNS

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A novel approach to the long-standing $^7$Li problem in Standard Big-Bang Nucleosynthesis (SBBN) is proposed via extensions beyond the Scale-Invariant Vacuum (SIV) framework. This work introduces the broader concept of Reparametrization-Invariant Scaling Symmetry (RISS), which allows for deviations from thermal equilibrium between matter and radiation during the BBN epoch. These deviations alter the scaling of nuclear reaction rates over time.

By incorporating RISS effects into the PRIMAT code, the model achieves excellent fits to observed primordial abundances:

- $\chi^2 < 0.04$ for $^4$He, D/H, and $^3$He/D
- $\chi^2 \approx 1$ for $^7$Li/H

The key result is a suppression of $^7$Be production, the primary precursor to $^7$Li, without invoking new particles or physics beyond the Standard Model. The analysis contrasts SIV-consistent and RISS-favored scenarios, highlighting differences in baryon density ($\Omega_b$) and thermal scaling behavior.

These findings suggest that the $^7$Li anomaly may arise from modified thermodynamic laws in the early universe, with potential implications for nuclear astrophysics and plasma physics.

# Riemann–Hilbert Problems and Integrable Equations

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The Lecture is based on the paper [1]. The standard approach to integrable nonlinear evolution equations (NLEE) usually uses the following steps, see [2, 3, 4]:

1. Lax representation $[L,M]=0$;
2. construction of fundamental analytic solutions (FAS);
3. reducing the inverse scattering problem (ISP) to a Riemann-Hilbert problem (RHP)
$$\xi^+(x,t,\lambda)=\xi^-(x,t,\lambda)G(x,t\lambda)$$
on a contour $\Gamma$ with sewing function $G(x,t,\lambda)$.
4. soliton solutions and possible applications. Step 1 involves several assumptions: the choice of the Lie algebra $g$ underlying $L$, as well as its dependence on the spectral parameter, typically linear or quadratic in $\lambda$.

Our idea is to use another approach that substantially extends the classes of integrable NLEE. Its first advantage is that one can effectively use any polynomial dependence in both $L$ and $M$. We use the following steps [**?**]:

**A.** Start with canonically normalized RHP with predefined contour $\Gamma$, say $\Gamma=\mathbb{R}\cup i\mathbb{R}$
$$\xi^+(x,t,\lambda)=\xi^-(x,t,\lambda)G(x,t,\lambda),\qquad \lambda\in\mathbb{R}\cup i\mathbb{R};$$

**B.** Specify the $x$ and $t$ dependence of the sewing function defined on $\Gamma$, say by:
$$i\frac{\partial G}{\partial x}-\lambda^2[J,G(x,t,\lambda)],\qquad i\frac{\partial G}{\partial t}-\lambda^4[J,G(x,t,\lambda)];\qquad \lambda\in\mathbb{R}\cup i\mathbb{R},$$
where $J$ is a constant diagonal matrix.

**C.** Introduce convenient parametrization for the solutions $\xi^\pm(x,t,\lambda)=\exp(\mathcal{Q}(x,t,\lambda)$, where $\mathcal{Q}(x,t,\lambda)=\sum_{s=1}^\infty\lambda^{-s}Q_s(x,t)$, which is compatible with the canonical normalization of RHP.

**D.** This RHP gives rise to a Lax pair
$$L\psi=i\frac{\partial\psi}{\partial x}-U(x,t,\lambda)\psi(x,t,\lambda)=0,\qquad M\psi=i\frac{\partial\psi}{\partial t}-V(x,t,\lambda)\psi(x,t,\lambda)=0,$$
$$U(x,t,\lambda)=\left(\lambda^2\xi^\pm J\hat{\xi}^\pm(x,t,\lambda)\right)_+,\qquad V(x,t,\lambda)=\left(\lambda^4\xi^\pm J\hat{\xi}^\pm(x,t,\lambda)\right)_+,$$
where the subscript $+$ means the polynomial part in $\lambda$ of the corresponding expression. One can check that $U(x,t,\lambda)$ and $V(x,t,\lambda)$ are parametrized by by $Q_1$, $Q_2$ and their $x$-derivatives and give rise to a system of nonlinear evolution equations (NLEE) of nonlinear Schrödinger type.

**E.** Use Zakharov–Shabat dressing method to derive their soliton solutions. This requires correctly taking into account the symmetries of the RHP.

**G.** Define the resolvent of the Lax operator and use it to analyze its spectral properties.

These results may be generalized also to Lax pairs with more complicated contours, such as $\Gamma = \mathbb{R} \cup i\mathbb{R} \cup \mathbb{S}^1$, where $\mathbb{S}^1$ is the unit circle. Such contours are due to additional symmetries of the Lax pair involving mappings $\lambda \to \lambda^{-1}$. Examples of NLEE with such additional symmetries will be presented.

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# Spectral Embeddings and Multi-Metric Graph Transformations for Robust Path Computation in Large-Scale Service Provider Networks

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Service provider networks face the challenge of computing reliable and efficient paths under dynamic topology changes, fluctuating traffic demands, and strict service-level agreements (SLAs).

Traditional shortest path first (SPF) algorithms, while computationally efficient, often overlook global structural properties of the network and may yield suboptimal routing under high-load or failure scenarios.

In this paper, we present a multi-metric path computation framework that combines spectral embeddings with adjacency-based graph transformations to enhance routing robustness and diversity.The framework integrates four complementary approaches:

1. Baseline SPF for classical shortest path computation
2. Spectral Embedding–Based Routing using the Fiedler vector of the graph Laplacian to capture global connectivity patterns
3. Inverse Adjacency–Based Routing, which amplifies indirect connectivity and uncovers low-interference alternative routes
4. Squared Adjacency Transformation, incorporating two-hop neighborhood connectivity to improve path redundancy and reduce bottleneck risks

# Application of equivariant transformations in neural networks to classification of galaxy images

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We train a deep neural network with $SE(2)$ equivariant layers on galaxy images grouped in two classes - cigar-shaped and round. We use a steerable basis to reduce the memory and computational load produced by the lifting correlations at different orientations of the images from the training dataset. The trained model performs with 99% accuracy in classifying images from the test dataset.

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