



FOURTH WORKSHOP ON SOLITON THEORY, NONLINEAR DYNAMICS AND MACHINE LEARNING

Tsarevo, Bulgaria, August 24, 2026 – August 29, 2026

Book of Abstracts

Organized by:
Institute for Advanced Physical Studies
Trakia University, Stara Zagora

Supported by:



Contract KP-06-MFN/16 from 18.05.2026



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**Program of the
Fourth Workshop on Soliton Theory, Nonlinear Dynamics
and Machine Learning
August 24 – 29, 2026, Tsarevo, Bulgaria**

The online participation is accessible via <https://nbu-bg.zoom.us/j/85716039350>

August 24 – Arrival and registration

August 25 (Tuesday)

A. Stefanov	9.30 – 10.00	Opening
	10.00 – 10.45	Higher-dimensional integrable systems. Algebraic aspects and reductions.

Coffee break 10.45 – 11.00

V. Gerdjikov	11.00 – 11.45	MNLS and Generalized Fourier Transforms for Constant Boundary Conditions
Li Chuanzhong	11.45 – 12.30	Classification on line solitons of KP type integrable systems

Lunch

V. Videv	14.30 – 15.10	Characterization of Riemannian and Lorentzian Manifolds via the Jacobi Operator
N. Pirovski	15.10 – 15.40	Solitons and Spirals: Connections in Nonlinear Dynamics and Biology

Coffee break 15.40 – 16.00

General discussion	16.00 – 18.00	
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August 26 (Wednesday)

M. Horník	09.30 – 10.10	Design and Permanent Magnets Reduction of Linear Oscillatory Machine for Reciprocating Drives
A. Kajević	10.10 – 10.50	Transient and Steady-State Dynamic Modeling of Synchronous Machine: TSFEM and the Winding Function Method

Coffee break 10.50 – 11.10

T. Podušel	11.10 – 11.50	Universal Multi-Phase Inverter Platform for Advanced Motor Control
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Lunch

A. Kolev	14.00 – 14.30	Unsupervised Vibration Fault Detection Using Hilbert-Huang Transformation-Based Feature Extraction and Mahalanobis Analysis
L. Temelakiev	14.30 – 15.00	Forward-Collision and Lane-Departure Warning on Edge AI Accelerators: A Cross-Device Evaluation of Raspberry Pi 5 and Jetson Orin Nano Super.
V. Manolov	15.00 - 15.30	Beyond Pass Completion: Automated Detection of Line-Breaking Passes in Association Football Using Limb Tracking Data and Their 3D Perceptual Reconstruction and Visualization
M. Naydenov	15.30 - 16.00	Axion contribution to the mass-radius relation of neutron stars

Coffee break 16.00 – 16.20

General discussion	16.20 – 18.00	
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20:00 WORKSHOP DINNER (Tsarevo, 17 Cherkovna Str.)

August 27 (Thursday)

R. Ivanov	09.30 – 10.10	Negative and rational flows of the integrable hierarchies
L. Ivanova	10.10 – 10.40	Derivation of the Nonlinear Schrodinger Equation in the Intermediate Long Wave Regime

Coffee break 10.40 – 11.00

A. Pauna	11.00 – 11.40	Necessary and sufficient conditions for an equation to accept a given type of solutions
H. Vekov	11.40 – 12.10	Scalable Approximation of Laplacian Eigenmodes for Spectral Routing in Service-Provider Networks

Lunch

G. Grahovski	14.00 – 14.40	On the derivative nonlinear Schrödinger equation related to symmetric spaces
L. Markov	14.40 – 15.10	Some new results involving inequalities of Gronwall-Bellman type
R. Marinova	15.10 – 15.40	Soliton and Periodic Wave Solutions of the Modified Korteweg–de Vries Equation
M. Flamarion	15.40 – 16.10	Extreme Events in Soliton Turbulence in Nonintegrable Systems
V. Gueorguiev	16.10 – 16.50	Time Reparametrization Invariance and the Hubble Tension

Coffee break 16.50 – 17.10

General Discussion	17.10 – 18.00	
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August 28 (Friday)

A. Isar	09.30 – 10.10	Generation of quantum correlations in Gaussian open quantum systems
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Coffee break 10.10 – 10.30

T. Mihaescu	10.30 – 11.10	Gaussian entanglement and steering of strongly coupled bosonic modes in a common phase-sensitive environment
A. Cudreasov	11.10 – 11.50	Cooperative Effects in Two Dipole–Dipole Coupled Emitters Driven by a Moderately Strong Laser Field

Lunch

M. Melikuziev	14.00 – 14.40	Data-Driven Multi-Objective Optimization for Smart Urban Distribution Networks
G. Petrov	14.40 – 15.20	Methodology for the Migration and Acceleration of an Open-Source Semiconductor Bloch Equation Solver Using Multi-Core CPU and GPU/CUDA Computing

Coffee break 15.20 – 15.40

General Discussion	15.40 – 18.00	
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August 29 (Saturday)

S. Mishev	09.30 - 10.10	Machine Learning on Symmetric Spaces. Basics.
Y. Foteva	10.10 - 10.40	Machine Learning on Symmetric Spaces. Structure of a Neural Network.

Coffee break 10.40 – 11.00

N. Blajev	11.00 – 11.30	Training Bulgarian Language Embeddings from Scratch through Self-Supervised Learning and Contrastive Learning
K. Djounakov	11.30 – 12.00	Comparative Study of Static Heterogeneous Graph Neural Networks and XGBoost for Credit Card Fraud Detection
	12.00 – 12.20	Closing

Lunch

Transient and Steady-State Dynamic Modeling of Synchronous Machine: TSFEM and the Winding Function Method

Aldin Kajević and Gojko Joksimović

University of Montenegro- Faculty of Electrical Engineering

This paper presents a comparative dynamic modeling study of synchronous machine using the Time-Stepping Finite Element Method (TSFEM) and the Winding Function (WF) method. A 2D TSFEM model is developed in Altair Flux for a real 200 MW, two-pole synchronous turbo-generator. The main steps involved in building the finite element model—including geometric modeling, mesh generation, material property assignment, and circuit coupling—are concisely outlined. Dynamic simulations are executed across three key operating conditions: starting transients, steady-state operation, and a sudden three-phase short circuit initiated from rated load conditions. The exact same operational scenarios are simulated using a dynamic model based on the Winding Function Method. A comparative analysis of the simulation results from both approaches is presented, evaluating accuracy, transient response waveforms, and computational execution times. The results demonstrate that while TSFEM provides detailed field distributions, the WF model achieves high accuracy at significantly faster execution speeds, making it exceptionally well-suited for integration into machine learning frameworks.

A. S. Cudreasov and M. A. Macovei
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REFERENCES

[illegible]

Higher-dimensional integrable systems. Algebraic aspects and reductions

Aleksander Stefanov

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences;
Faculty of Mathematics and Informatics, Sofia University "St.Kliment Ohridski"

We examine integrable systems that can be related to a non-local Riemann-Hilbert problem, or in more general to a non-local DBAR problem. We outline a method for constructing the ideal in the ring of differential operators generated by such a problem, which in turn allows the construction of new integrable systems. We also consider reductions.

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Unsupervised Vibration Fault Detection Using Hilbert-Huang Transformation-Based Feature Extraction and Mahalanobis Analysis

Atanas Kolev¹, Yassen Gorbounov¹, Pavol Rafajdus², Gojko Joksimović³

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Feature extraction plays a central role in vibration-based fault detection, particularly when labeled data are unavailable. This study investigates the use of the Hilbert-Huang Transform as an unsupervised feature extraction method for rolling bearing vibration signals. Statistical and frequency-domain features are extracted from the Intrinsic Mode Functions obtained through Empirical Mode Decomposition and the Marginal Hilbert Spectrum, followed by Principal Component Analysis for dimensionality reduction and Mahalanobis distance for anomaly scoring. The proposed workflow is evaluated on a publicly available rolling bearing dataset containing healthy and faulty operating conditions. Under the investigated experimental conditions, the extracted features provide a clear separation between normal and faulty samples, resulting in high fault detection performance, without supervised model training. While these findings suggest that the extracted features effectively capture fault-related information in the evaluated dataset, they are limited to a single experimental study. Future work will examine the proposed feature extraction approach across additional datasets, operating conditions, and fault types.

GENERATION OF QUANTUM CORRELATIONS IN GAUSSIAN OPEN QUANTUM SYSTEMS

Alina Stoica and Aurelian Isar

Department of Theoretical Physics, National Institute of Physics and Nuclear
Engineering, Bucharest-Magurele, Romania

Hellinger geometric quantum discord and interferometric power are investigated for an open system consisting of two bosonic modes interacting with four different environments: vacuum, squeezed vacuum, thermal and squeezed thermal environment, taking the initial state of the system either a unimodal squeezed state, which presents no correlations, or a squeezed vacuum state. The time evolution of the correlations is investigated in the framework of the theory of open systems, based on completely positive quantum dynamical semigroups, using the Gorini-Kossakowski-Lindblad-Sudarshan master equation. We show that both quantum correlations can be generated from an initial factorized state. For the squeezed vacuum environment, the competition between the influences produced by the squeezing of the initial state and the squeezing of the environment leads to the decreasing, and even destruction of the correlations, or to their enhancement. In the case of an initial squeezed vacuum state in interaction with the squeezed vacuum environment, for specific conditions one can achieve decoherence-free states, while for the interaction with a thermal environment there exist regimes of enhancing or decreasing the considered correlations, depending on the parameters of the open system and environment [1].

REFERENCES

- [1] Alina Stoica, Aurelian Isar, European Physical Journal Plus 141 (2026), 559.

Classification on line solitons of KP type integrable systems

Chuanzhong Li

Shandong University of Science and Technology, China

In this talk, we will give a classification of the regular soliton solutions of the KP hierarchy, referred to as the KP solitons, under the Gel'fand-Dickey reductions in terms of the permutation of the symmetric group. As an example, we show that the regular soliton solutions of the (good) Boussinesq equation as the 3-reduction can have at most one resonant soliton. In particular, we show that the non-crossing permutation gives the regularity condition for the soliton solutions.

On the derivative nonlinear Schrödinger equation related to symmetric spaces

Georgi Grahovski

Department of Mathematical Sciences, University of Essex, Wivenhoe Park,
Colchester CO4 3SQ (UK)

We will present multi-component generalizations of derivative nonlinear Schrödinger (DNLS) type of equations having quadratic bundle Lax pairs related to $\mathbb{Z}_2$ -graded Lie algebras and **A.III** symmetric spaces. The Jost solutions and the minimal set of scattering data for the case of local and nonlocal reductions are constructed.

Furthermore, the fundamental analytic solutions (FAS) are constructed and the spectral properties of the associated Lax operators are briefly discussed. The Riemann-Hilbert problem (RHP) for the multi-component generalizations of DNLS equation of Kaup-Newell (KN) and Gerdjikov-Ivanov (GI) types is derived.

The infinite set of integrals of motion for these models is briefly described at the end. Based on a joint work with Vladimir Gerdjikov and Rossen Ivanov.

Methodology for the Migration and Acceleration of an Open-Source Semiconductor Bloch Equation Solver Using Multi-Core CPU and GPU/CUDA Computing

Georgi Petrov

New Bulgarian University

Legacy scientific simulation software often combines mature numerical algorithms implemented primarily in Fortran and C, posing significant challenges for migration to modern heterogeneous computing architectures. While these legacy implementations provide validated computational models, they frequently suffer from limited scalability, maintainability, and inefficient utilization of contemporary multi-core CPU and GPU platforms. This paper presents a methodology for the migration and acceleration of an open-source Semiconductor Bloch Equation (SBE) numerical solver through a sequence of incremental modernization stages while preserving strict numerical consistency and precision.

The proposed methodology begins with an architectural analysis of system performance, followed by the identification of performance-critical computational kernels implemented in Fortran. These legacy numerical modules are subsequently optimized for multi-core CPU execution through progressive parallelization and performance tuning. Each optimization stage is validated against the original implementation to ensure numerical equivalence. After establishing a reliable parallel CPU implementation, the computationally intensive integration kernels are gradually migrated to GPU execution using a Python-based CUDA framework, enabling heterogeneous CPU/GPU computing while maintaining compatibility with the original software architecture. Continuous numerical verification is performed throughout the migration process to guarantee reproducibility and precision. The modernized computational framework is further extended with a low-rank matrix reduction methodology based on Random Matrix Theory and the Marchenko–Pastur criterion, applied to optimize the core interaction matrices. This extension reduces computational complexity while preserving the structural components of the processed data, providing a robust foundation for advanced algorithmic acceleration.

Experimental evaluation demonstrates that the migrated hybrid implementation preserves numerical accuracy with a relative L2 error approaching machine precision while achieving substantial computational acceleration compared with the original implementation. The proposed methodology shows that legacy scientific simulation software can be systematically modernized through incremental migration, continuous validation, heterogeneous CPU/GPU acceleration, and algorithmic optimization without requiring a complete software redesign.

The source code and implementation details are publicly available at: <https://github.com/gwttgwtt/pySBE>

Engineering contribution: A systematic methodological framework for migrating legacy scientific simulation software to heterogeneous CPU/GPU computing architectures while preserving numerical reproducibility.

Scientific/Methodological contribution: Integration of Random Matrix Theory and the Marchenko–Pastur criterion for low-rank reduction of dense system matrices, enabling computational complexity reduction while maintaining high numerical accuracy.

Scalable Approximation of Laplacian Eigenmodes for Spectral Routing in Service-Provider Networks

Hristo Vekov

Faculty of Mathematics and Informatics, Sofia University "St.Kliment Ohridski"

Spectral routing uses the global structure of a network graph to identify topological bottlenecks, enhance path diversity, and complement conventional shortest-path computation. Its scalability is constrained by the computational cost of obtaining the low-frequency eigenpairs of the graph Laplacian for large and structurally complex networks. Extending our previous framework for robust path computation based on spectral embeddings and multi-metric graph transformations, this research paper implements scalable framework for approximating the first N th lowest-frequency eigenpairs of the symmetric network Laplacian. The proposed approach integrates Chebyshev and Legendre polynomial approximations of a heat-kernel spectral response with block subspace iteration. The method is evaluated across 100 independently seeded trials, each involving a distinct weighted topology containing approximately 20,000 edges. These topologies combine structured local connectivity with randomly generated long-range edges that are usually encountered in Border Gateway Protocol (BGP) route-reflector peerings and Dynamic Multipoint VPN (DMVPN) hub-and-spoke topologies. Independent trials are executed concurrently using Loky backend to increase experimental throughput. On average eigenvector cosine similarity is approximately 0.99 when compared to ARPACK reference solutions. The results demonstrate that polynomial-filtered eigensolvers can accurately recover the Laplacian eigenvectors required to support diverse spectral path-selection algorithms in dynamically evolving service-provider networks.

SOME NEW RESULTS INVOLVING INEQUALITIES OF GRONWALL–BELLMAN TYPE

Lubomir Markov
Barry University, USA

The interesting subject of a certain class of widely applicable inequalities begins with Gronwall, who in 1919 proves the following result [4, p. 293]:

THEOREM 1. *Let $x(t)$ be a continuous real-valued function on $J = [t_0, t_0 + h]$, and suppose that on J*

$$0 \leq x(t) \leq \int_{t_0}^t (Mx(s) + A) ds,$$

where M and A are nonnegative constants. Then $0 \leq x(t) \leq Ah \exp(Mh)$ on J .

In 1943, R. Bellman proves [2] an extension of Gronwall's result:

THEOREM 2. *Let $x(t)$ and $k(t)$ be real-valued nonnegative continuous functions for $t \geq t_0$. If $a \geq 0$ is a constant, and*

$$x(t) \leq a + \int_{t_0}^t k(s)x(s) ds, \quad t \geq t_0,$$

then

$$x(t) \leq a \exp \left(\int_{t_0}^t k(s) ds \right), \quad t \geq t_0.$$

Inequalities of this type, of which a wealth of examples exists [1, 3], have numerous applications to the study of uniqueness and stability properties of differential equations. It is the purpose of this presentation to introduce some (believed to be new) inequalities of Gronwall-Bellman type.

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 - [2] R. Bellman, *Stability Theory of Differential Equations*, McGraw Hill, New York, 1953.
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Intermediate Long Wave Regime

Lyudmila Ivanova

Technological University Dublin, Ireland

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Comparative Study of Static Heterogeneous Graph Neural Networks and XGBoost for Credit Card Fraud Detection

Krassimir Djounakov^{1,2}, Apostol Mushmov² and Stoyan Mishev¹

¹New Bulgarian University

²Management Financial Group

Credit card fraud detection involves highly imbalanced data, evolving behavioral patterns, and complex relationships between transactions and associated entities. This thesis investigates whether a heterogeneous graph neural network (GNN) can outperform a well-tuned XGBoost baseline under comparable feature conditions.

Using the IEEE-CIS Fraud Detection dataset, transactions and related entities, including cards, addresses, devices, and email domains, are represented as a heterogeneous graph. A temporally consistent experimental framework is introduced in which cumulative train, validation, and test graphs are constructed chronologically to prevent structural information leakage. A staged feature-ablation approach is then used to assess the contribution of transaction features, entity information, and graph structure.

The results show that GNN performance improves substantially as richer information is introduced; however, the best graph-based model achieves a PR-AUC of 0.390, compared with 0.539 for XGBoost. The hypothesis that a static heterogeneous graph representation provides superior fraud classification is therefore not confirmed.

The findings highlight the limitations of static message passing when predictive signal is dominated by transaction-level features and entity nodes remain relatively sparse. The study provides a leakage-aware framework for fair GNN–XGBoost comparison and motivates future work on dynamic, edge-aware graph models incorporating temporal and transactional relationships.

Design and Permanent Magnets Reduction of Linear Oscillatory Machine for Reciprocating Drives

Matuš Horník¹, Pavol Rafajdus¹, Hao Chen², Yassen Gorbounov³, Gojko Joksimović⁴

¹ University of Zilina, Department of Power System and Electric Drives, Zilina, Slovakia

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The growing demand for efficient and sustainable electrical drives has increased interest in reducing the use of rare-earth permanent magnets while maintaining machine performance. This paper presents the design and optimization of a stator-permanent-magnet transverse-flux linear oscillatory machine for reciprocating compressor applications. The machine was designed and analyzed using three-dimensional finite element analysis, and its geometry was optimized within predefined dimensional constraints. Electromagnetic performance and thermal behavior were evaluated to verify the feasibility of the proposed topology and the safe operating temperatures of the permanent magnets. Finally, the mechanical design and manufacturing documentation for a prototype were developed. The results demonstrate that the permanent magnet volume can be significantly reduced while maintaining the required thrust force, providing a compact and efficient solution for reciprocating compressor drives.

Extreme Events in Soliton Turbulence in Nonintegrable Systems

Marcelo V. Flamarion¹, Efim Pelinovsky² and Ekaterina Didenkulova²

¹Pontifical Catholic University of Peru

²HSE University, Russia

We study the emergence of extreme wave events, often termed rogue or freak waves, within nonlinear dispersive media. Their formation is investigated through Monte Carlo simulations of soliton ensembles with randomly distributed amplitudes, corresponding to the regime of soliton gas, in the Schamel, modified Korteweg–de Vries (mKdV), and related equations. The resulting turbulent wave fields are analyzed through their principal statistical characteristics, including spectral properties, higher-order moments, and probability density functions.

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-Objective Optimization for Smart Urban Distribution Networks

Mirkomil Melikuziev

Tashkent State Technical University named after Islam Karimov, Tashkent, Uzbekistan

This study presents a data-driven multi-objective optimization approach for the planning and operation of smart urban distribution networks. The proposed framework simultaneously considers technical, economic, and reliability criteria, including power losses, capital expenditures, voltage constraints, transformer loading, and reliability indices. A multi-objective optimization procedure is applied to determine efficient network configurations and identify compromise solutions among conflicting objectives. The approach integrates distribution-network operating data with optimization techniques to support the optimal placement and sizing of key network elements. The obtained results demonstrate that coordinated optimization can reduce power losses and investment costs while improving voltage profiles and overall supply reliability. The proposed approach can support decision-making for the planning and modernization of urban distribution networks under increasing load demand and the integration of distributed energy resources.

Axion contribution to the mass-radius relation of neutron stars

Momchil Naydenov

Faculty of Physics, Sofia University "St.Kliment Ohridski"

An effective Lagrangian for the interaction between a pseudo-scalar (axion-like) field and massive fermions is considered. At high density and non-zero temperature axions can be produced through bremsstrahlung. If the axion-neutron interaction is greater than a certain value we can have a mean free path smaller than the size of a neutron star. The influence of such trapped axions on the mass-radius function of the neutron star is investigated by solving numerically the Tolman-Oppenheimer-Volkoff equations. We show that causality limits the applicability of the trapping regime. We find specific ranges of central densities in a neutron star and temperatures for which axions give a conspicuous contribution to the neutron-star mass. For other densities and temperatures axions do not have a significant effect on the structure of a neutron star. Since we use an effective approach, our results can be easily applied to specific realistic axion models extending our toy model, if interactions due to additional Standard Model particles (e.g. neutrinos) are included.

Training Bulgarian Language Embeddings from Scratch through Self-Supervised Learning and Contrastive Learning

Nikola Blajev

New Bulgarian University

Natural language processing (NLP) enables computers to analyse human language by converting text into mathematical representations.

Embedding models encode tokens, sentences, and documents as dense vectors, placing semantically similar content close together in a shared vector space.

Most Bulgarian NLP applications rely on multilingual or English-centred models, which may not fully capture the language’s morphology, syntax, and lexical structure.

This project develops a Bulgarian-centred embedding model trained entirely from scratch on a large, cleaned corpus of Bulgarian text.

A specialised subword tokenizer and compact Transformer encoder are trained using self-supervised masked language modelling to acquire lexical, semantic, and syntactic knowledge.

The model is then refined through contrastive learning, bringing related sentences closer together and pushing unrelated sentences apart in the embedding space.

The resulting representations are evaluated through semantic similarity, classification, clustering, semantic search, paraphrase detection, and text deduplication.

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Solitons and Spirals: Connections in Nonlinear Dynamics and Biology

Nikola Pirovski

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IFS-BAS

There is a deep and fascinating connection between solitons and spirals. Solitons are remarkably stable, self-sustaining wave packets that maintain their shape and speed over long distances thanks to a delicate balance between nonlinear effects and dispersion. Spirals, in turn, often provide the natural geometric framework in which these solitons can stabilize, interact, and give rise to rich spatiotemporal patterns.

In biological systems this relationship becomes especially meaningful. Many tissues and cell networks behave as excitable media, where electrical, chemical, or calcium signals propagate according to nonlinear rules. In such environments, spiral waves are frequently viewed as dissipative solitons — robust, rotating structures that sustain themselves over time.

A well-known example is the Belousov-Zhabotinsky reaction, which produces concentric and spiral waves that display classic soliton-like properties. These patterns have direct counterparts in living systems, such as the spiral cAMP waves that coordinate the collective aggregation of *Dictyostelium discoideum* amoebae.

According to the Spiral Theory of the Human Body (STHB), the spiral serves as a fundamental organizing principle across multiple levels — from the DNA double helix and the cochlea to the spine and the cardiovascular system. The human body itself can be seen as an integrated spiral system with a central attractor.

In the heart, spiral waves (often called rotors) act as dissipative solitons and are a major driver of dangerous arrhythmias such as ventricular tachycardia and fibrillation. These waves can anchor to tissue heterogeneities like scars or fibrotic zones and maintain chaotic electrical activity. Modern optogenetic techniques now make it possible to gently drift and terminate these spirals. Similar spiral soliton-like waves appear in the brain during cortical spreading depression — the phenomenon underlying migraine aura — and in epileptic seizures. In reproductive biology, spiral calcium waves following fertilization function as dissipative solitons that coordinate cortical granule release and prevent polyspermy.

The same principles extend to large-scale atmospheric phenomena: the persistent spiral arms of hurricanes, polar vortices that can trigger extreme weather, and the rotating updrafts in supercell storms.

From a theoretical perspective, these spiral waves are elegantly described by reaction-diffusion equations such as the FitzHugh-Nagumo model. They naturally emerge as stable vortices around phase singularities. Integrating soliton theory with the Spiral Theory of the Human Body offers fresh insight into how nonlinear dynamics shape both health and disease.

In summary, solitons and spirals are far more than beautiful geometric forms — they are fundamental dynamic structures that nature uses to organize life, from cellular signals to heart rhythms and beyond.

Necessary and sufficient conditions for an equation to accept a given type of solutions

Corina N. Babalic, Radu Constantinescu, Alina Pauna

University of Craiova, Romania

The paper aims to outline the necessary and sufficient conditions for an equation to admit solutions of a predefined mathematical form. We will consider a generalized form of a nonlinear differential equation with polynomial coefficients and identify the subclasses of equations that admit Riccati-type solutions. The necessary condition will concern the polynomial degrees of the coefficients, while the sufficient condition will consist of satisfying certain compatibility conditions among the parameters appearing in the equation's coefficients.

To make the analysis concrete, we will particularize the general equation for several models of practical interest, such as the well-known Burgers, Korteweg–de Vries and nonlinear Schrödinger equations.

Negative and rational flows of the integrable hierarchies

Rossen Ivanov

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The soliton hierarchies are characterised by a pair of Lax operators (L and M), L is usually polynomial in the spectral parameter. For the AKNS hierarchy (and its generalisations) for example, L is linear in the spectral parameter. The equations arising from the negative flows (when M involves negative powers of the spectral parameter) are usually equations in non-evolutionary form. These include important examples such as the Camassa-Holm [1] and Qiao equations, when L is in the $\mathfrak{sl}(2)$ algebra, and the Degasperis-Procesi equation when L is in the $\mathfrak{sl}(3)$ algebra (with the corresponding reductions, [2]). When L is quadratic (the so-called quadratic pencil or quadratic bundle) there are negative flows as well, the best-known example being the Fokas-Lenells equation (and its extensions on Hermitian symmetric spaces).

We discuss other examples [3], related to the quadratic bundle and the Heisenberg hierarchy, which is gauge equivalent to the AKNS hierarchy as well as examples where M is a rational function of the spectral parameter [4].

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This presentation introduces an inverse problem based approach for computing solitary and periodic wave solutions of the modified Korteweg–de Vries equation. Key solution parameters are identified through a variational formulation that transforms the resulting ill-posed inverse problem into a well-posed minimization problem suitable for numerical computation. Numerical experiments demonstrate the accuracy, stability, and robustness of the proposed methods for both solitary and periodic waves. In particular, the proposed framework provides a new computational means of constructing periodic solutions that are difficult to obtain using existing methods.

Machine Learning on Symmetric Spaces. Basics.

Stoyan Mishev^{1,2} and Yoana Foteva¹

New Bulgarian University, Sofia, Bulgaria
Institute for Advanced Physical Studies, Bulgaria

Neural networks are a complex non-linear transformations in which non-linearities are introduced artificially. Recently [1,2,3] a mathematical framework in which data is mapped to a Riemannian space equivalent to a non-compact coset manifold which is a symmetric manifold. This structure represents a layer in a "neural networks" and transformations between such layers in non-linear by their nature. We discuss also related topics such as the Tits-Satake universality classes and the Paint group.

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Gaussian entanglement and steering of strongly coupled bosonic modes in a common phase-sensitive environment

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We investigate the dynamics of two strongly interacting bosonic modes coupled to a common phase-sensitive Gaussian environment, focusing on the generation and preservation of entanglement and Einstein–Podolsky–Rosen steering. In the strong-coupling regime, the dissipative dynamics is described using a global master-equation approach formulated in the normal-mode basis [1,2]. For a common reservoir, the transformation reveals a particularly simple structure: only one collective normal mode interacts with the environment, while the second mode remains decoupled from the bath. This bright–dark mode structure strongly influences the long-time behaviour of quantum correlations in the original bosonic modes. Using the covariance-matrix formalism, we analyze the dependence of Gaussian entanglement and directional steering on the intermode coupling and on the properties of the phase-sensitive reservoir.

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Universal Multi-Phase Inverter Platform for Advanced Motor Control

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Multi-phase electric drives provide improved reliability, fault tolerance, and enhanced control flexibility compared to conventional three-phase systems. This paper presents the design and implementation of a universal multi-phase inverter (UMPI) intended for research and development in the field of multi-phase motor control. The inverter is designed with emphasis on universality, modularity, and compatibility with industrial standards, enabling configurable phase numbers and operation with various power-stage topologies. The functionality and versatility of the developed platform were validated through simulation studies in MATLAB/Simulink. The inverter was evaluated using five- and nine-phase induction motor models. In addition to verifying stable multi-phase operation, the influence of third-harmonic current injection on electromagnetic torque production was investigated. The simulation results confirm that the proposed inverter satisfies the requirements for multi-phase drive applications and enables implementation of advanced control strategies.

Time Reparametrization Invariance and the Hubble Tension

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The discrepancy between the value of the Hubble constant H_0 obtained by local measurements and the one deduced from the Cosmic Microwave Background (CMB) observations has been a persistent issue for modern cosmology and is known as the Hubble Tension. Without a strong guiding principle the space of possibility to explore is limitless. As such, it has been an arena of fruitless explorations of exotic physics using models with new particles and fields. A recent 2026 paper by Courbin and Maeder have demonstrated that the tension can be resolved with the Scale Invariant Vacuum (SIV) paradigm. In this paper, a mathematically equivalent but physically different idea about the time Re-parameterization Invariant Scaling Symmetry (RISS) is utilized to build a model that addresses the Hubble Tension without the need of new particles or fields. The time dependent units rescaling $(dt, dr) \rightarrow \lambda(t)(dt, dr)$ is almost exact within the modern treatment of the CMB phenomenon. While the corresponding equivalent symmetry, based on the Weyl transformation $(a, N) \rightarrow (\lambda(t)a, N)$ imposes augmentation of the Friedmann-Lemaître-Robertson-Walker (FLRW) equations for cosmology as done in the SIV paradigm. The Hubble Tension is then recognized to be due to difference in the unit of time during the decoupling era and the current era since the corresponding gravitational potentials are different.

Characterization of Riemannian and Lorentzian Manifolds via the Jacobi Operator

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In the present paper we characterize certain classes of Riemannian and Lorentzian manifolds by means of the characteristic coefficients of the Jacobi operator. We relate our study to the three types of gravitational fields defined by A. Z. Petrov, which are solutions of Einstein's equation from the General Theory of Relativity, and by means of the Jacobi operator we characterize spaces of constant sectional curvature, decomposable spaces, and spaces of maximal mobility.

MNLS and Generalized Fourier Transforms for Constant Boundary Conditions

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The talk is dedicated to the memory of Professor David Kaup, whose results deeply affected our scientific investigations.

The inverse problem for an integrable system of 3 coupled equations of Nonlinear Schrödinger type with constant boundary conditions (CBC) is studied. The system has been proposed previously [1] as an integrable 1-dimensional model for the Bose-Einstein condensate of particles with spin 1 in the absence of an external field. The model requires CBC, since they are related to the condensate average particle density.

The Lax pair of the system is associated to the symmetric C.I space $SP(4)/U(2)$ in the special case of CBC. The fundamental analytic solutions of the Lax operator L are constructed and two minimal sets $\mathfrak{T}_i$, $i = 1, 2$ of scattering data are identified. A generalization of the Wronskian relations, taking into account the CBC, allows the construction of two sets of “squared solutions” of L . The completeness of the “squared solutions” is proven, which allows for an explicit expansions of the derivative of the potential $Q_x(x, t)$ of L and the variation $\delta Q(x, t)$ over the “squared solutions”. The expansion coefficients of these expansions are directly related to the sets $\mathfrak{T}_i$ and $\delta\mathfrak{T}_i$. Thus, the mapping between the potential $Q(x, t)$ and the minimal sets of scattering data can be viewed as a Generalized Fourier Transform and the “squared solutions” of L - as generalized exponents. A properly chosen recursion operator, for which the “squared solutions” are their eigenfunctions, allows for the description of the related integrable hierarchy of equations with CBC.

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Beyond Pass Completion: Automated Detection of Line-Breaking Passes in Association Football Using Limb Tracking Data and Their 3D Perceptual Reconstruction and Visualization

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This paper presents an automated framework for detecting line-breaking passes in association football, addressing a long-standing limitation whereby post-match analysis reduces such actions to a binary completion metric despite their considerable tactical significance and disproportionate contribution to goals. The proposed pipeline synchronizes tracking and event data and applies constrained K-means clustering to dynamically classify a defending team’s outfield players into formation lines, from which passes that intersect a line while sufficiently advancing the ball toward goal are identified as line-breaking. Empirical results show that such passes nearly double the probability of a goal occurring (3.78% versus 1.93%), motivating the introduction of the Pass Advantage Score, a novel metric quantifying a receiver’s positional advantage based on the zone of reception and proximity to the nearest defender. Building on this foundation, the framework is extended through the incorporation of 3D limb-tracking data, enabling a more granular, four-question evaluation of passing decisions spanning classification, perception, decision quality, and outcome. This extension is realized through pitch-control modelling, a passing-option classifier informed by player pose and head orientation, and a first-person virtual-reality reconstruction of match sequences. Together, these components form a unified two- and three-dimensional framework that advances beyond geometric pass detection toward a perceptually grounded assessment of passing decisions, offering coaching staff and performance analysts a richer and more interpretable tool for post-match analysis.

Machine Learning on Symmetric Spaces. Structure of a Neural Network.

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Hyperbolic space is a Riemannian manifold with well-known advantages for embedding hierarchical data. Thanks to a classical works by Élie Cartan, it is also metrically equivalent to a solvable Lie group. In this talk we present a hyperbolic architecture for deep learning that takes advantage of this double nature. Each layer in the neural network is a combination of group homomorphisms and isometries acting on solvable coordinates. We provide the necessary background on Lie theory – Lie algebras, solvable algebras and Cartan subalgebras – and explain how this algebraic structure determines the coordinates used in the network, from embedding data to performing hyperbolic classification at the output. We also discuss the architecture’s performance on benchmark tasks and open directions for testing its core motivation on hierarchical data.

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